

## Critical dynamics in liquids with long-range forces

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The critical dynamics at phase transitions in liquids with long-range forces, behaving like  $r^{-3-\sigma}$ , is considered. The critical dimension in these systems is  $d_c = 2 + \sigma$ , leading to nonclassical behavior of the transport coefficients at  $d = 3$  in the region  $1 < \sigma < 1.5$  and classical behavior in static properties. We calculate the transport coefficients for pure liquids and mixtures in the hydrodynamic region. The critical exponent of the order-parameter Onsager coefficient  $x_\lambda$  and of the shear viscosity  $x_\eta$  fulfill  $x_\lambda + x_\eta = \sigma - 1$  in  $d = 3$ . In the crossover region from critical to "classical" dynamical behavior for  $\sigma \rightarrow 1$  the dynamical transient exponents go to zero and nonasymptotic effects become dominant in the Kawasaki amplitude. A direct measurement of  $\sigma$  in the dynamics is proposed.

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### I. INTRODUCTION

It is not yet clear whether in ionic solutions "classical" critical behavior due to long-range interactions has been observed or one just has seen a crossover to Ising-like asymptotic critical behavior. Recent experiments gave support for classical static critical behavior at the consolute points in ionic solutions [1-5]. Since the consolute point and the gas-liquid critical point (called plait point in a mixture) lie in the same universality class, similar behavior can be expected for these transitions in pure liquids and liquid mixtures. It is suspected that the Coulomb interaction (to some extent screened) or other molecular multipole interactions lead to an effective interaction of long range of the type  $r^{-3-\sigma}$  with  $0 < \sigma < 2$ , but the specific type of the effective interaction remains unclear. It has been shown in [6] that in the case of  $0 < \sigma < 1.5$  the static exponents are classical,  $\gamma = 1$ ,  $\nu = \frac{1}{\sigma}$ , and  $\eta = 2 - \sigma$  fulfilling the usual scaling laws leading to  $\beta = 1$  and  $\alpha = 0$ . Therefore static measurements of  $\gamma$ ,  $\beta$ , or  $\alpha$  give no further information about the parameter  $\sigma$  governing the range of the effective interaction. However, it is important to note that the classical critical exponents  $\nu$  and  $\eta$  do depend on the parameter  $\sigma$ . Thus direct measurements of the static exponents  $\nu$  or  $\eta$  may give information on the type of the effective interaction. However, this seems to be not so easy since the correlation length  $\xi$  appears in the static susceptibility as  $\xi^{-\sigma}$ . Therefore, in a neutron scattering experiment of the  $k$ -dependent static susceptibility as a function of the temperature  $t$  ( $t = |T - T_c|/T_c$ ) the parameter  $\sigma$  drops out. However, the  $k$  dependence of the susceptibility would provide one with  $\sigma$  (at  $T_c$  the susceptibility is proportional to  $k^{-\sigma}$ ). To our knowledge no such experiments have been performed so far.

Light scattering measurements of dynamical quantities such as the mass diffusion at the consolute point have

been performed [3-5]. No theory has been presented so far for the dynamical critical behavior in these solutions and we therefore extend recent work [7,8] on the critical behavior in liquid mixtures with short-range interactions to the case of ionic solutions with long-range interaction. We shall see that the parameter  $\sigma$  enters the dynamical quantities and should in principle be observable in these experiments. We find that the dynamical behavior is nonclassical for  $1 < \sigma \leq 1.5$  and different from that in the universality class of the short-range interacting systems. For  $\sigma < 1$  the Onsager coefficients (OCs) are nonsingular, for  $\sigma > 1$  two OCs show singular behavior, the one for the shear viscosity  $L_j$ , which diverges as  $(T - T_c)^{-\frac{\eta}{\sigma}}$ , and the one for the order parameter (the entropy density at the gas-liquid critical point or the concentration fluctuation at the consolute point)  $L_{11}$ , which diverges as  $(T - T_c)^{-\frac{\lambda}{\sigma}}$ . Therefore, measurements of the temperature dependence of the transport coefficients in the hydrodynamic region can also give direct information on the parameter  $\sigma$ . Moreover, the critical exponents increase almost linearly from  $\sigma = 1$  to their values in liquids with short-range interactions at  $\sigma = 2$ .

### II. MODEL

Let us start with the Ginzburg-Landau-Wilson Hamiltonian  $\mathcal{H}$  for the order parameter  $\phi$ , which is the entropy density  $\sigma$  for the gas-liquid critical point in the pure fluid as well as for the plait point in the mixture

$$\phi = N_A^{\frac{1}{2}}(\sigma - b_\sigma) \quad (1)$$

(there should be no confusion with the parameter  $\sigma$  of the interaction) or the concentration fluctuation  $c$  for the consolute point

$$\phi = N_A^{\frac{1}{2}}(c - b_c) \quad (2)$$

with  $N_A$  Avogadro's number;  $b_\sigma$  and  $b_c$  are static constants, by which third order terms in the Hamiltonian have been transformed away. Then we end up with the Hamiltonian in Fourier space [6]

$$\begin{aligned} \mathcal{H}_\phi &= \int d^d k \frac{1}{2} (\tau + k^\sigma) \phi_k \phi_{-k} \\ &+ \int d^d k \int d^d k' \int d^d k'' \frac{u_0}{4!} \phi_k \phi_{k'} \phi_{k''} \phi_{-k-k'-k''}. \end{aligned} \quad (3)$$

Although in the classical region  $\sigma < 1.5$  the fourth order coupling is irrelevant, this term has to be taken into account in the region of spontaneous order. To this Hamiltonian we add Gaussian terms for the other variables to be considered in dynamics, namely, at the plait point the concentration fluctuation at constant entropy density

$$q = N_A^{\frac{1}{2}} \left\{ c - \left( \frac{\partial c}{\partial \sigma} \right)_{P\Delta} (\sigma - b_\sigma) \right\}, \quad (4)$$

where  $\Delta$  is the difference in chemical potential of the two components of the mixture. At the consolute point we must add the entropy density at constant concentration

$$q = N_A^{\frac{1}{2}} \left\{ \sigma - \left( \frac{\partial \sigma}{\partial c} \right)_{PT} (c - b_c) \right\} \quad (5)$$

and in both cases the transverse current  $\mathbf{j}$

$$\mathcal{H}_{q,j} = \int d^d k \frac{1}{2} \{ a q(k) q(-k) + a_j \mathbf{j}(k) \mathbf{j}(-k) \}. \quad (6)$$

Nonlinear couplings between the order parameter and the second thermodynamic variable are irrelevant since we are in the classical regime of the statics. So the total Hamiltonian reads

$$\mathcal{H} = \mathcal{H}_\phi + \mathcal{H}_{q,j}. \quad (7)$$

We note again that for calculations in the nonordered region  $T > T_c$  this Hamiltonian is purely Gaussian.

The order-parameter susceptibility diverges as  $t^{-1}$ , but the susceptibilities involving the other thermodynamic variable are finite (since the specific heat exponent  $\alpha = 0$ ), thus for the pure liquid ( $\mathcal{R}$  is the gas constant)

$$\langle \phi \phi \rangle_c(k=0) = \frac{\mathcal{R}T}{\rho} \left( \frac{\partial \sigma}{\partial T} \right)_P \quad (8)$$

and at the plait point

$$\langle \phi \phi \rangle_c(k=0) = \frac{\mathcal{R}T}{\rho} \left( \frac{\partial \sigma}{\partial T} \right)_{P\Delta}, \quad (9)$$

$$\langle qq \rangle_c(k=0) = \frac{\mathcal{R}T}{\rho} \left( \frac{\partial c}{\partial \Delta} \right)_{P\sigma}.$$

Standard thermodynamic manipulations show that away from azeotropic criticality we have  $\left( \frac{\partial c}{\partial \Delta} \right)_{P\sigma} \sim t^{-1}$  [9,10]. At the consolute point we have

$$\langle \phi \phi \rangle_c = \frac{\mathcal{R}T}{\rho} \left( \frac{\partial c}{\partial \Delta} \right)_{PT}, \quad \langle qq \rangle_c = \frac{\mathcal{R}T}{\rho} \left( \frac{\partial \sigma}{\partial T} \right)_{Pc}. \quad (10)$$

Let us now turn to the dynamics, which is our main concern. We must consider the densities of the conserved hydrodynamic variables, which also contain the order parameter. We then recover model  $H$  for the pure fluid or model  $H'$  for mixtures [11]

$$\dot{\phi} = L_{011} \nabla^2 \frac{\delta \mathcal{H}}{\delta \phi} + L_{012} \nabla^2 \frac{\delta \mathcal{H}}{\delta q} - g_0 (\nabla_i \phi) \frac{\delta \mathcal{H}}{\delta j_i} + \Theta_\phi, \quad (11)$$

$$\dot{q} = L_{012} \nabla^2 \frac{\delta \mathcal{H}}{\delta \phi} + L_{022} \nabla^2 \frac{\delta \mathcal{H}}{\delta q} - g_0 (\nabla_i q) \frac{\delta \mathcal{H}}{\delta j_i} + \Theta_q, \quad (12)$$

$$\dot{\mathbf{j}} = \mathcal{T} (L_{0j} \nabla^2 \frac{\delta \mathcal{H}}{\delta \mathbf{j}} + g_0 (\nabla \phi) \frac{\delta \mathcal{H}}{\delta \phi} + g_0 (\nabla q) \frac{\delta \mathcal{H}}{\delta q} + \Theta_j). \quad (13)$$

The matrix  $L_0$  and the coefficient  $L_{0j}$  contain the independent OCs, which are related to the fluctuating forces  $\Theta_i$  via Einstein relations. The mode coupling is  $g_0 = RT/N_A^{\frac{1}{2}}$  and  $\mathcal{T}$  is the projector transverse to the  $\mathbf{k}$  direction. Note that model  $H$  contains only the equations for  $\phi$  and  $\mathbf{j}$  without the coupling to the second variable  $q$ . In the following we treat model  $H'$  within the field theoretic renormalization group formalism. Proceeding in the standard way we have to renormalize the dynamical vertex functions. Note that all static renormalization constants are 1 for  $\sigma \leq 1.5$ . Then only the renormalization factors  $Z_{L_{ij}}$ ,  $i, j = 1, 2$ , and  $Z_{L_j}$  may remain different from unity. The renormalization and regularization is performed as in [12], which means that we have to set the irrelevant parameters (the parameters with negative naive dimension) equal to zero in extracting the poles from the vertex functions. From the structure of the graph contributions to the renormalization we find that only  $L_{11}$  and  $L_j$  renormalize, which leads to a singular behavior in the corresponding OCs.  $L_{12}$  and  $L_{22}$  stay unrenormalized and merely enter as constant parameters.

We introduce dynamical parameters for the model by the diffusion time ratio  $w_0$  and the mode coupling  $f_0$

$$w_0 = \frac{L_{012}}{\sqrt{L_{011} L_{022}}}, \quad f_0 = \frac{g_0}{\sqrt{L_{011} L_{0j}}}. \quad (14)$$

They renormalize as

$$w_0 = w Z_{L_{11}}^{-\frac{1}{2}}, \quad f_0 = f \mu^\epsilon Z_{L_{11}}^{-\frac{1}{2}} Z_{L_j}^{-\frac{1}{2}}, \quad (15)$$

with  $\epsilon = 2 + \sigma - d$  and a reference wave number  $\mu$ . Thus we see that the dynamical critical dimension above which the mode coupling terms are irrelevant is given by

$$d_c^{\text{dynamic}} = 2 + \sigma. \quad (16)$$

This may be compared with  $d_c^{\text{static}} = 2\sigma$ , which is always below the dynamical critical dimension. Therefore in the region  $1 < \sigma < 1.5$  the statics is classical whereas the dynamics shows nonclassical critical behavior. For  $0 < \sigma < 1$  the OCs remain uncritical.

The change of the renormalized parameters under renormalization is described by the flow equations, which contain the  $\zeta$  functions  $\zeta_i = \mu \frac{\partial \ln Z_i^{-1}}{\partial \mu}$ . No static  $\zeta$  function appears since we are above the static critical dimension. In one-loop order the  $\zeta$  functions read

$$\zeta_{L_{11}} = -\frac{1+\sigma}{2+\sigma}f^2, \quad \zeta_{L_j} = -\frac{\sigma^2(1+\sigma)}{12(4+\sigma)(2+\sigma)}\frac{f^2}{1-w^2}. \quad (17)$$

In the asymptotic region the parameters reach their fixed point values and the values of the  $\zeta$  functions determine the critical exponents. From the flow equations at  $d = 3$  ( $l$  is related to the inverse correlation length)

$$l\frac{dw}{dl} = -\frac{1}{2}w\zeta_{L_{11}}, \quad l\frac{df}{dl} = -\frac{1}{2}f(\sigma - 1 + \zeta_{L_{11}} + \zeta_{L_j}), \quad (18)$$

the fixed point values are found to be  $w^* = 0$  and

$$f^{*2} = \frac{12(4+\sigma)(2+\sigma)}{(48+12\sigma+\sigma^2)(1+\sigma)}(\sigma-1). \quad (19)$$

For the pure fluid  $w(l) \equiv 0$  has to be put in for the  $\zeta$  functions; as a consequence the same fixed point values are obtained. Thus the asymptotic critical behavior is the same in all three cases of phase transitions in the liquids. We define renormalized OCs as (note that no static parts appear in these definitions)

$$\tilde{L}_{11}(l) = L_{11}(l) \exp \int_1^l \zeta_{L_{11}} \frac{dl'}{l'}, \quad (20)$$

$$\tilde{L}_j(l) = L_j(l) \exp \int_1^l \zeta_{L_j} \frac{dl'}{l'}$$

with the one-loop expressions

$$L_{11}(l) = L_{11} \left[ 1 - \left( \frac{1}{2+\sigma} \right)^2 f^2(l) \right], \quad (21)$$

$$L_j(l) = L_j \left( 1 - \frac{\sigma(24+46\sigma+17\sigma^2+\sigma^3)}{24(2+\sigma)^2(4+\sigma)^2} \frac{f^2(l)}{1-w^2(l)} \right). \quad (22)$$

### III. TRANSPORT COEFFICIENTS

The transport coefficients (TCs) are identified by comparing the model equations with the usual hydrodynamic equations. The vertex functions by which they are expressed can be split into a static and a genuine dynamical part as a result of fluctuation dissipation theorems. This allows us to treat static and dynamic effects separately. For the temperature dependence of the static functions we can insert the experimental results, whereas for the dynamic parameters we can use the flow equations. These are calculated in one-loop order.

In the pure fluid the singular transport coefficients are the thermal conductivity and the shear viscosity

$$\kappa(t) = \tilde{L}_{11}(t), \quad \bar{\eta}(t) = \tilde{L}_j(t) \quad (23)$$

with the susceptibility  $\chi(t)$  given by the specific heat at constant pressure. Asymptotically the TCs reach the behavior

$$\kappa \sim t^{-\frac{x_\lambda}{\sigma}}, \quad \bar{\eta} \sim t^{-\frac{x_\eta}{\sigma}}. \quad (24)$$

The exponents in one-loop order are found to be (remember  $\epsilon = 2 + \sigma - d$ ; thus for  $d = 3$  we have simply  $\epsilon = \sigma - 1$ )

$$x_\lambda = \frac{12(4+\sigma)}{48+12\sigma+\sigma^2}(\sigma-1), \quad (25)$$

$$x_\eta = \frac{\sigma^2}{48+12\sigma+\sigma^2}(\sigma-1).$$

If we take, e.g.,  $\sigma = 1.5$ , then for  $d = 3$ ,  $x_\lambda = 0.484$ ; this value decreases to zero at  $\sigma = 1$ . So as expected we find weaker singularities in the OCs compared to the short-range interacting case mainly due to the shorter distance from the dynamic critical dimension (see Fig. 1).

From (18) one finds in the region where the stable fixed point has a nonzero fixed point value for  $f$ , the exact relation for the critical exponents

$$x_\lambda + x_\eta = \sigma - 1. \quad (26)$$

Since  $x_\eta$  is small, the approximation  $x_\lambda = \sigma - 1$  may be used as a first step in the experimental analysis. Thus measurements of the asymptotic temperature dependence of the thermal conductivity in a pure fluid would directly determine the exponent  $\sigma$ .

In the mixture at the plait point singular behavior appears in the isothermal diffusion constant  $D$ , in the thermal conductivity in the absence of mass flow  $\kappa$ , in the thermal diffusion ratio  $k_T$ , and in the shear viscosity  $\eta$

$$D(t) = \frac{1}{\chi_c(t)} [L_{22} + a^2 \tilde{L}_{11}(t) + 2aL_{12}], \quad (27)$$

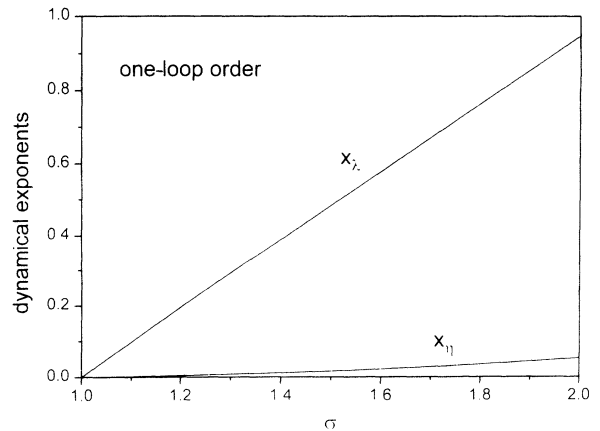


FIG. 1. The dynamical critical exponents of the order-parameter Onsager coefficient  $x_\lambda$  and of the shear viscosity  $x_\eta$  as functions of the range  $\sigma$  of the interaction [see Eqs. (25)].

$$\kappa(t) = \frac{1}{\chi_c(t)D(t)} [\tilde{L}_{11}(t)L_{22} - L_{12}^2], \quad (28)$$

$$\begin{aligned} k_T(t) &= \frac{T}{D(t)} [L_{12} + a\tilde{L}_{11}(t)] - T \left( \frac{\partial c}{\partial T} \right)_{p\Delta} \\ &= -\frac{T}{aD(t)} (L_{22} + aL_{12}) - T \left( \frac{\partial c}{\partial T} \right)_{p\sigma}, \end{aligned} \quad (29)$$

and  $\bar{\eta}(t) = \tilde{L}_j(t)$ , with the static functions  $\chi_c(t) = \left( \frac{\partial c}{\partial \Delta} \right)_{pT}$  and  $a = \left( \frac{\partial c}{\partial \sigma} \right)_{p\Delta}$ . Asymptotically we obtain for the behavior of the TCs

$$D \sim t^{1-\frac{\sigma}{\lambda}}, \quad \kappa = \kappa_c, \quad k_T \sim t^{-1+\frac{\sigma}{\lambda}}. \quad (30)$$

Large ratios of  $\frac{L_{22}+2aL_{12}}{\sigma^2}$  can mask the asymptotic behavior as is observed in short-range interacting mixtures [13].

The temperature dependence of the OCs is described in our theory by the flow equations (18), the expressions (20) and (21), the connection of the flow parameter to the temperature distance  $t = t_\Delta$  at constant chemical potential  $\Delta$ , and the relation of  $t_c$  (the temperature distance at constant concentration) to  $t_\Delta$ .

At the consolute point the concentration fluctuation instead of the entropy fluctuation is the order parameter, leading also to another relation of the TCs to the OCs, namely,

$$D(t) = \frac{\tilde{L}_{11}(t)}{\chi_c(t)}, \quad \kappa(t) = L_{22} - \frac{L_{12}^2}{\tilde{L}_{11}(t)}, \quad k_T(t) = T \frac{L_{12}}{D(t)}. \quad (31)$$

The asymptotic behavior is the same as at the plait point, but nonasymptotic effects play a much smaller role in this case.

The Kawasaki amplitude [14] in  $d = 3$  is defined for pure liquids by the following ratio involving the thermal diffusion  $D_T = \kappa/\chi$  and the shear viscosity

$$R_\sigma(t) = \frac{D_T(t)\bar{\eta}(t)\xi(t)}{k_B T} \quad (32)$$

and asymptotically reaches a universal value, since from (18) we have  $x_\lambda + x_\eta = \sigma - 1$ . We calculated  $R$  in one-loop order. Its asymptotic value relative to the value in liquids with short-range interaction reads at  $d = 3$

$$\frac{R_\sigma^*}{R_2^*} = \frac{1}{\sigma - 1} r(\sigma), \quad (33)$$

$$\begin{aligned} r(\sigma) &= \frac{2(48 + 12\sigma + \sigma^2)(1 + \sigma)}{19(4 + \sigma)(2 + \sigma)} \frac{1 - \frac{12(\sigma-1)(4+\sigma)}{(48+12\sigma+\sigma^2)(2+\sigma)}}{1 - \frac{3}{38}} \\ &\times \frac{1 - \frac{(\sigma-1)\sigma(24+46\sigma+17\sigma^2+\sigma^3)}{2(48+12\sigma+\sigma^2)(2+\sigma)(4+\sigma)(1+\sigma)}}{1 - \frac{2}{57}}. \end{aligned} \quad (34)$$

In the range  $2 \geq \sigma \geq 1$   $r(\sigma)$  decreases from  $r(2) = 1$  only by 4% and one therefore can approximate its value by 1.

The amplitude is formally divergent when one approaches the dynamical critical dimension. This is an artifact of the asymptotic expression. One has to consider in the case of a small fixed point value of  $f$  and because of small slow transients the nonasymptotic expression of the amplitude [15]. The dependence of this nonasymptotic amplitude on  $\sigma$  and on the flow parameter  $l$  is shown in Fig. 2 for the pure liquid [ $w(l) \equiv 0$  in Eqs. (18)] in the lowest approximation

$$\frac{R_\sigma(l)}{R_2^*} = \frac{24}{19f(l)^2}. \quad (35)$$

At the consolute point because of the different order parameter the amplitude is defined with the mass diffusion  $D$  instead of the thermal diffusion  $D_T$

$$R_\sigma(t) = \frac{D(t)\bar{\eta}(t)\xi(t)}{k_B T} \quad (36)$$

and takes on the same asymptotic value.

#### IV. DISCUSSION

We have calculated the critical behavior of long-range interacting ionic solutions near the critical point. Comparison with the results for the case of short-range interactions in one-loop approximation shows that the genuine dynamical singular behavior is weakened, as expected for long-range forces. First and most important the dynamical critical dimension is lowered and therefore its distance to  $d = 3$  is smaller, and second the prefactors in the  $\epsilon$  expansion of the dynamical critical exponents itself become slightly  $\sigma$  dependent. Note that our results for the dynamical quantities are also valid in the region  $1.5 < \sigma < 2$  since the neglected contributions from the statics appear only in two-loop order (in fact one has to cope with the

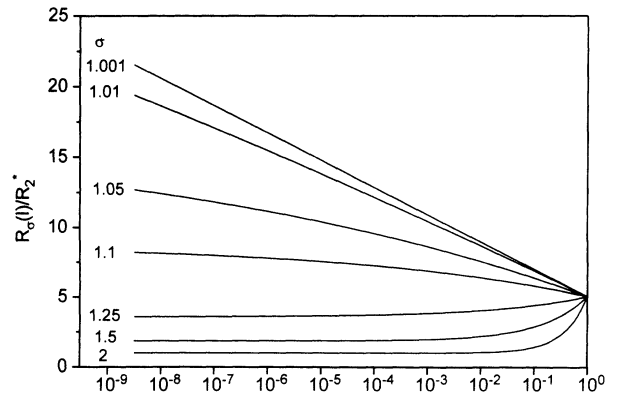


FIG. 2. The ratio of the nonasymptotic Kawasaki amplitude for the long-range interacting liquid to the asymptotic value of this amplitude for the short-range interacting liquid as a function of the flow parameter  $l$  [see Eq. (35) with Eqs. (18)] for different values of  $\sigma$ . The initial values are  $f(0) = 0.5$  and  $w(0) = 0$ . For values of  $\sigma$  near 1 the effect of the slow transients proportional to  $\sigma - 1$  can be seen.

problem that static and dynamic have different critical dimensions).

It is interesting to look for the consequences of our results on the interpretation of light scattering experiments. Since these cover the hydrodynamic  $k\xi \ll 1$  as well as the critical  $k\xi \gg 1$  region, we have to consider the asymptotic scaling function  $\bar{\Omega}_\sigma(k\xi)$ , commonly referred to as the "Kawasaki function" for  $\sigma = 2$  [14]. Let us for this purpose approximate  $x_\lambda$  by  $\sigma - 1$  (which corresponds to the usual result of mode coupling theory) and  $x_\eta$  by 0. The order-parameter dispersion can be written as (in pure liquids and in mixtures at the consolute point)

$$\omega_\phi(k, \xi) = \frac{L_{11}(k, \xi)}{\chi(k, \xi)} k^2 \quad (37)$$

involving the order-parameter OC  $L_{11}$  and the order-parameter susceptibility  $\chi$ . The dynamical critical exponent  $z$  is then defined by the order-parameter frequency at  $T_c$

$$\omega_\phi(k, \infty) \sim k^z. \quad (38)$$

Since  $\chi(k, \xi) = \frac{k^{-\sigma}}{1+(k\xi)^{-\sigma}}$  and  $L_{11}(k, \infty) \sim k^{-x_\lambda}$  we find  $z = 2 + \sigma - x_\lambda = 3$  independent of  $\sigma$ . We may also write

$$\omega_\phi(k, \xi) = R_\sigma^* \frac{k^2 k_B T}{\bar{\eta} \xi} \bar{\Omega}_\sigma(k\xi) \quad (39)$$

defining the scaling function  $\bar{\Omega}_\sigma$  similar to the case of short-range interacting liquids. For  $\sigma = 2$  and within mode coupling theory [14] the function equals

$$\bar{\Omega}_2(y) = \frac{3}{4} y^{-2} [1 + y^2 + (y^3 - y^{-1}) \arctan(y)]. \quad (40)$$

The limits of the Kawasaki function

$$\bar{\Omega}_\sigma(0) = 1, \quad \bar{\Omega}_\sigma(k\xi \rightarrow \infty) \sim k\xi \quad (41)$$

are independent of  $\sigma$ . Of course, the scaling function depends on  $\sigma$ , but we expect this dependence to be weak (this will be checked in the future). Lacking the explicit form of  $\bar{\Omega}_\sigma$  one may in a first analysis of an experiment make the following approximation: Replace  $\bar{\Omega}_\sigma$  by  $\bar{\Omega}_2$ , take for the ratio of the Kawasaki amplitudes (33) its value with  $r(\sigma) = 1$ , and use for  $R_2^*$  the mode coupling value. Then (42) simplifies to

$$\frac{\omega_\phi(k, \xi)}{k^2} = \frac{1}{\sigma - 1} \frac{k_B T}{6\pi\bar{\eta}\xi} \bar{\Omega}_2(k\xi). \quad (42)$$

From this relation we suggest to extract the temperature dependence of  $\xi$ , which then leads to the exponent  $\nu = \frac{1}{\sigma}$ .  $\sigma$  has to be treated as a parameter which should be chosen "optimally," whatever this means. One choice may be such that the region in which the asymptotic power law is observed is maximal. We should remark, however, that near  $\sigma = 1$  these asymptotic relations have to be replaced by their nonasymptotic counterparts. A calculation of these functions has to be done in future. We also note that there may be long-range order interactions with several values of  $\sigma$ . The asymptotics is governed by the smallest value of  $\sigma$  but crossovers from one value of  $\sigma$  to another may be possible. This may make the interpretation of the experimental results difficult.

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